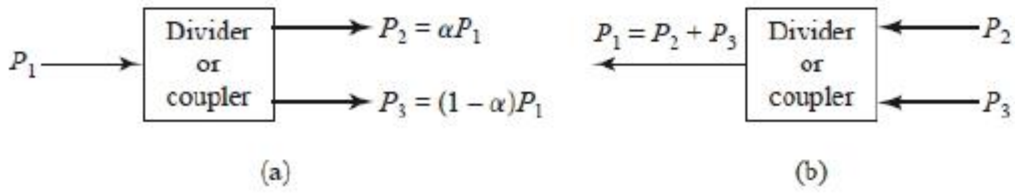
S-PARAMETERS

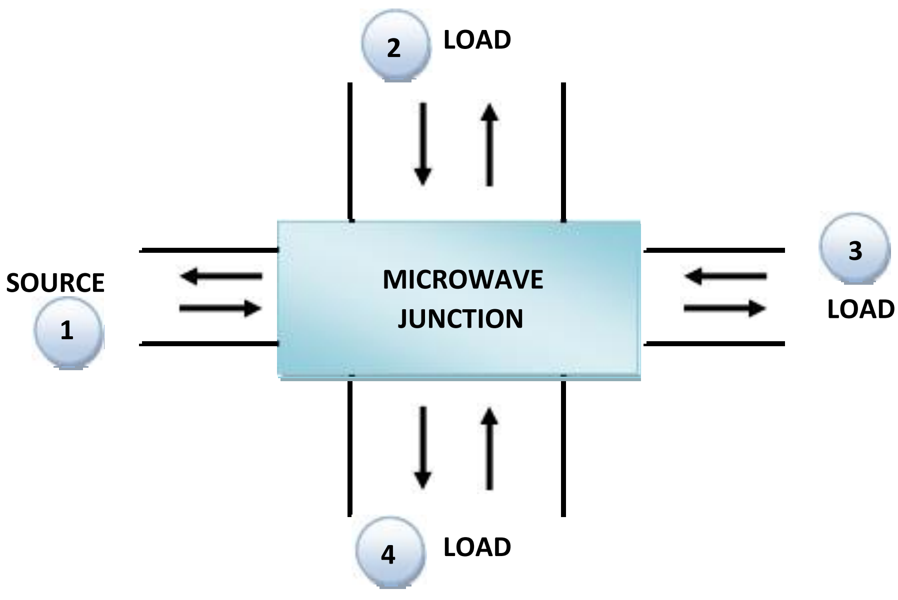
1. INTRODUCTION:

- Power dividers and directional couplers are passive microwave components used for power division or power combining, as illustrated in Figure 7.1.



Power division and combining. (a) Power division. (b) Power combining.

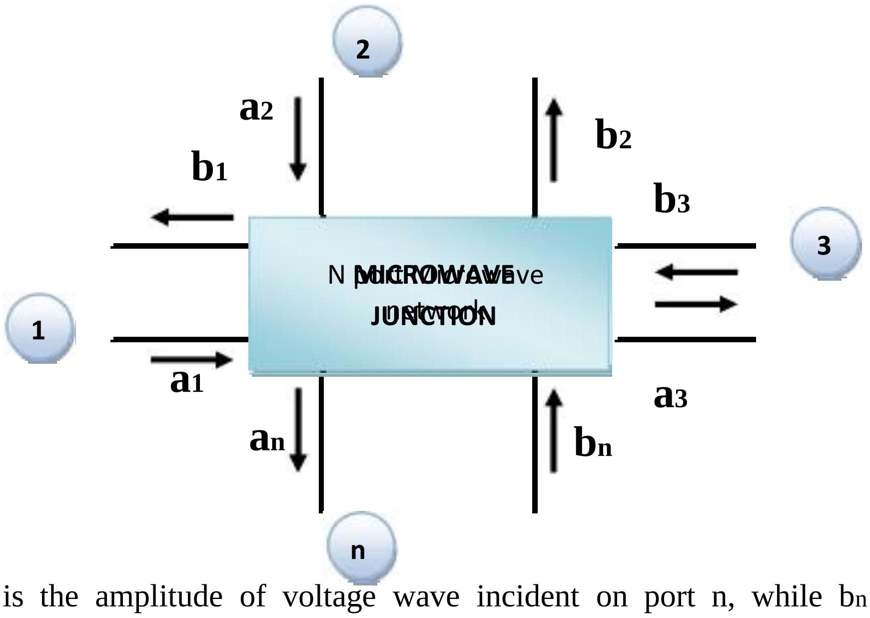
- In power division, an input signal is divided into two (or more) output signals of lesser power, while a power combiner accepts two or more input signals and combines them at an output port.
- The coupler or divider may have three ports, four ports, or more, and may be (ideally) lossless.
- Three-port networks take the form of T-junctions and other power dividers, while four-port networks take the form of directional couplers and hybrids.
- Power dividers usually provide in-phase output signals with an equal power division ratio (3 dB), but unequal power division ratios are also possible.
- Directional couplers can be designed for arbitrary power division, while hybrid junctions usually have equal power division. Hybrid junctions have either a 90◦ or a 180◦ phase shift between the output ports.
- A microwave junction is an interconnection of two or more microwave components as shown in figure 2 below.



2. THE SCATTERING MATRIX:

- The low frequency circuits can be represented in two port networks and characterized by their parameters i.e. impedances, admittances, voltage gain, current gain, etc. All these parameters relate total voltages and currents at the two ports.
- In addition, a practical problem exists when trying to measure voltages and currents at microwave frequencies because direct measurements usually involve the magnitude (inferred from power) and phase of a wave traveling in a given direction or of a standing wave. Thus, equivalent voltages and currents, and the related impedance and admittance matrices, become somewhat of an abstraction when dealing with high-frequency networks.
- So at microwave frequency the logical variables used are travelling waves with associated powers, rather than total voltages and total currents. These logical variables are called as **S- parameters.**
- So in microwave analysis, the power relationship between the various ports of microwave junction is defined in terms of parameters, called as **S-parameters or scattering parameters.**
- As the microwave junction is a multiport junction, the power relationship between the various ports are defined in terms of matrix form, and called as **S matrix**, which a square matrix giving all the power combinations between the input port and output ports.
- Equipments are not readily available to measure total voltage and current at the ports of the network for microwave range. Also it is difficult to achieve short and open circuits on a large bandwidth of frequencies.

- The relationship between the scattering matrix and input/output powers at different ports can be obtained for N port microwave junction as shown in figure2.



- a_n is the amplitude of voltage wave incident on port n, while b_n is the amplitude of the reflected voltage wave from port n.

- If the ports are not properly matched with the junction, there will be reflection from junction, back towards the ports.
- The scattering matrix or [S] matrix is defined in relation to these incident and reflected voltage waves as

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Reflected waves or output	S-matrix	Input or Incident waves

$$[b] = [S][a] \quad \ldots\ldots\ldots\ldots\ldots (1)$$

The specific element of S-matrix is $S_{ij} = \frac{b_i}{a_j}$ i.e. scattering coefficient due to input at j^{th} port and output taken from i^{th} port.

- The incident waves on all ports except the j^{th} port are set to zero, i.e. all ports should be terminated in matched load to avoid reflections.
- Thus, S_{ii} is reflection coefficient at the port 1, when the same port is exited with incident waves, and rests of the ports are terminated in matched loads.

Properties of S-matrix

1. Scattering matrix is always a square matrix of order n x n.

2. $[S^*][S]' = [I]$.
   i.e. S matrix is unit matrix,
   I=identity matrix of same order as that of S,
   S^* = Complex conjugate.

3. Scattering matrix posses property of symmetry,
   i.e. $S_{ij} = S_{ji}$

4. $\sum_{i=1}^{n} S_{ij} S_{ik}^* = 0, for\ j \neq k, \quad j, k = 1, 2, 3, \ldots\ldots n.$
   i.e. sum of products between any row and column with complex conjugate of any other row or column is zero.

5. If any port, moved away from the junction by a distance of βd, then the coefficients of S_{ij} involving that particular port will be multiplied by the factor $e^{-j\beta d}$.

Properties of S-matrix for Reciprocal and Lossless Network

The impedance and admittance matrices are symmetric for the reciprocal network and imaginary for the lossless networks. Similarly scattering matrix i.e. [S] matrix for a reciprocal network is symmetric, and unitary for lossless network.

Any two port network which will satisfy the following condition is called as reciprocal network.

$$\frac{V_1}{I_2} = \frac{V_2}{I_1} = Z_{12} = Z_{21}, \dots\dots\dots(2)$$

$V_1, V_2 = voltage\ applied\ to\ port\ 1, and\ 2$
$I_1, I_2 = current\ applied\ to\ port\ 1, and\ 2$
Z_{12}, Z_{21}
$= Corresponds\ to\ symmetric\ impedance\ or\ transfer\ impedance\ between\ port\ 1\ and\ 2.$

Similarly for **reciprocal** type of network, S matrix id symmetric i.e. $S_{ij} = S_{ji}$

Also this condition can be written in terms of $[S] = [S]^t$ **………(3)**,
$[S]^t$ is the transpose fo matrix $[S]$

If network is **lossless**, then the real power delivered to the network, must be **zero.**
For **lossless** network [S] matrix is **unitary.** Any matrix which will satisfy

$$[S]^t[S^*] = [I] \dots\dots\dots(4)$$

is called as unitary matrix. This equation can be modified as

$$[S]^* = \{[S]^t\}^{-1} \dots\dots\dots(5)$$

The equation (5) can be written in summation form as,

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = \delta_{ij}, \text{ for all } i, j,$$

…………. (6)

where $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$, is the Kronecker delta symbol.

Thus , if i = j,

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1,$$

……………. (7)

If $i \neq j$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = 0, \text{ for } i \neq j.$$

…………… (8)

In words, equation (7) states that the dot product of any column of [S] with the conjugate of that same column gives unity, while equation (8) states that the dot product of any column with the conjugate of a different column gives zero (because columns are ortho normal).

Example:

A two-port network is known to have the following scattering matrix:

$$[S] = \begin{bmatrix} 0.15\angle 0^\circ & 0.85\angle -45^\circ \\ 0.85\angle 45^\circ & 0.2\angle 0^\circ \end{bmatrix}$$

Determine if the network is reciprocal and lossless. If port 2 is terminated with a matched load, what is the return loss seen at port 1? If port 2 is terminated with a short circuit, what is the return loss seen at port 1?

Solution

Because $[S]$ is not symmetric, the network is not reciprocal. To be lossless, the scattering parameters must satisfy (4.53). Taking the first column [$i = 1$ in (7))] gives

$$|S_{11}|^2 + |S_{21}|^2 = (0.15)^2 + (0.85)^2 = 0.745 \neq 1,$$

so the network is not lossless.

When port 2 is terminated with a matched load, the reflection coefficient seen at port 1 is $\Gamma = S_{11} = 0.15$. So the return loss is

$$\text{RL} = -20 \log |\Gamma| = -20 \log(0.15) = 16.5 \text{ dB}.$$

When port 2 is terminated with a short circuit, the reflection coefficient seen at port 1 can be found as follows. From the definition of the scattering matrix and the fact that $b_2 = -a_2$ (for short circuit at port 2), we can write as

$$b_1 = S_{11} a_1 + S_{12} a_2 = S_{11} a_1 - S_{12} b_2 \ldots (i)$$

$$b_2 = S_{21} a_1 - S_{22} a_2 = S_{21} a_1 - S_{22} b_2 \ldots (ii)$$

The equation (ii) gives

$$b_2 = \frac{S_{21}}{1 + S_{22}} a_1$$

Dividing equation (i) by a_1, and using the above result gives the reflection coefficient seen at port 1 as,

$$\Gamma = \frac{b_1}{a_1} = S_{11} - \frac{S_{12} b_2}{a_1} = S_{11} - \frac{S_{12} S_{21}}{1 + S_{22}}$$

$$= 0.15 - \frac{(0.85\angle -45^\circ)(0.85\angle 45^\circ)}{1 + 0.2} = -0.452.$$

So the return loss is $\text{RL} = -20 \log |\Gamma| = -20 \log(0.452) = 6.9$ dB. ■

An important point to understand about scattering parameters is that the reflection coefficient looking into port n is not equal to S_{nn} unless all other ports are matched (this is illustrated in the above example). Similarly, the transmission coefficient from port m to port n is not equal to S_{nm} unless all other ports are matched. The scattering parameters of a network are properties only of the network itself (assuming the network is linear), and are defined under the condition that all ports are matched. Changing the terminations or excitations of a network does not change its scattering parameters, but may change the reflection coefficient seen at a given port, or the transmission coefficient between two ports.

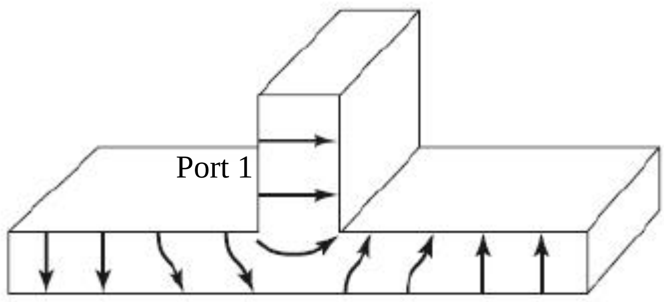
WAVEGUIDE TEES :

Waveguide Tees and couplers are junctions or networks having three or more ports. Waveguide Tees are used for the purpose of connecting a branch section of waveguide in series or parallel with the main waveguide.

3. E-Plane TEE JUNCTION (Series Tee):

Port 3

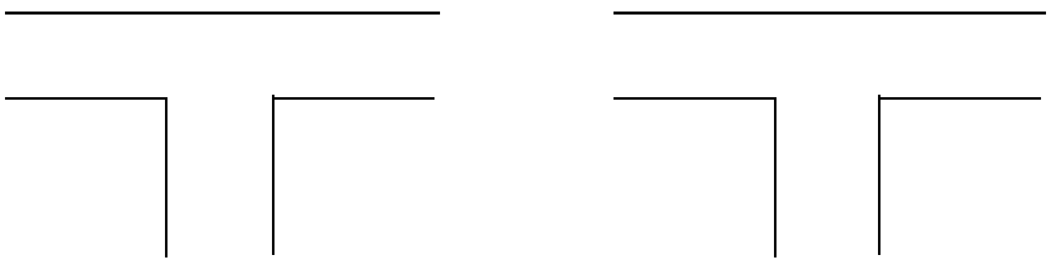
E arm (side arm)



Port 2

Collinear arm

- As shown in figure above is an E-plane Tee junction, as it is an intersection of three waveguides in the form of alphabet T. Port 1 and 2 are collinear arms while port 3 is the E arm, which is along the broader dimensions of waveguides.
- The T junction is used for power division or power combining.
- E-plane Tee is a voltage or series junction – symmetrical about the central arm so that the signal to be split up (or signals to be combined are taken from it) is fed from it. However, the problem has more complexities than it appears superficially. This is because some form of unwanted reflections occurs and it is essentially to provide some sort of impedance matching to minimize reflections. In fact, E-plane tee may themselves be used for impedance matching purposes in a manner similar to the short circuited transmission line stub; where a short circuit at any point is produced by means of a movable piston.
- When the dominant mode TE_{10} is made to propagated through port 3, the outputs from port 1 and 2 will be at the same amplitude but phase shifted by 180° with respect to each other. This 180° phase shift is occurring between port 1 and 2 is due to the change in electric field lines.
- As E-plane tee is symmetrical about the central arm, power coming out from port 3, is proportional to the difference between the power entering from port 1 and 2. When power entering from port 1 and 2 are in phase opposition, then maximum power comes out of port 3.



- Since it is a three port junction the scattering matrix can be derived as follows:
  1. [S] Matrix of order 3 x 3.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

  2. The Scattering coefficients are

$$\boldsymbol{S_{23} = -S_{13} \ldots\ldots\ldots\ldots (10)}$$

     As the waves coming out of the port 1 and 2 of the collinear arm will be opposite phase and in same magnitude. Negative sign indicates phase difference.

  3. If the port 3 is perfectly matched to the junction $\boldsymbol{S_{33} = 0 \ldots\ldots\ldots\ldots (11)}$
  4. For symmetric property $\boldsymbol{S_{ij} = S_{ji}}$

$$\boldsymbol{\therefore S_{12} = S_{21}\ ;\ S_{13} = S_{31};\ S_{23} = S_{32} \ldots\ldots\ldots\ldots (12)}$$

     with the above properties, [S] becomes,

$$\boldsymbol{[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (13)}$$

  5. From unitary property, $\boldsymbol{[S][S]^* = [I]}$

$$\boldsymbol{\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\boldsymbol{R_1C_1: \quad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \ldots\ldots\ldots (14)}$$

$$\boldsymbol{R_2C_2: \quad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \ldots\ldots\ldots (15)}$$

$$\boldsymbol{R_3C_3: \quad 0 + |S_{13}|^2 + |S_{13}|^2 = 1 \ldots\ldots\ldots (16)}$$

$$\boldsymbol{R_3C_1: \quad S_{13}.S_{11}^* - S_{13}.S_{12}^* = 1 \ldots\ldots\ldots (17)}$$

     From equations (14), and (15), we get

$$\boldsymbol{S_{11} = S_{22} \ldots\ldots\ldots\ldots\ldots\ldots.. (18)}$$

     From equation (16), $\boldsymbol{S_{13} = \frac{1}{\sqrt{2}} \ldots\ldots\ldots\ldots\ldots\ldots (19)}$

     From equation (17), $\boldsymbol{S_{13}(S_{11}^* - S_{12}^*) = 0}$

$$\boldsymbol{but\ S_{13} \neq 0 \ \therefore (S_{11}^* - S_{12}^*) = 0}$$

$$\boldsymbol{\therefore S_{11}^* = S_{12}^* = S_{11} = S_{12} = S_{22} \ldots\ldots\ldots\ldots\ldots\ldots (20)}$$

     Using these values from equation 18, 19 and 20 in equation 14,

$$\boldsymbol{|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1}$$

$$\boldsymbol{\therefore S_{11} = \frac{1}{2} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)}$$

Substituting the values of equation 19,20 and 21, the [S] matrix of equation 13 becomes

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

We know that, $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\therefore b_1 = \frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

$$\therefore b_2 = \frac{1}{2}a_1 + \frac{1}{2}a_2 - \frac{1}{\sqrt{2}}a_3 \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

$$\therefore b_3 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2 \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Case 1: Input is given at port 3 and no inputs at port 1 and 2, $a_3 \neq 0, a_1 = a_2 = 0$.

From equation 23, $b_1 = \frac{1}{\sqrt{2}}a_3$

From equation 24, $b_2 = -\frac{1}{\sqrt{2}}a_3$

From equation 25, $b_3 = 0$

Case 2: Input is given at port 1 and port 2, and no input at port 3, $a_3 = 0, a_1 = a_2 = a$.

From equation 23, $b_1 = \frac{a}{2} + \frac{a}{2}$

From equation 24, $b_2 = \frac{a}{2} + \frac{a}{2}$

From equation 25, $b_3 = 0$

Case 3: Input is given at port 1 and no input at port 2 and port 3, $a_1 \neq 0, a_3 = a_2 = 0$.

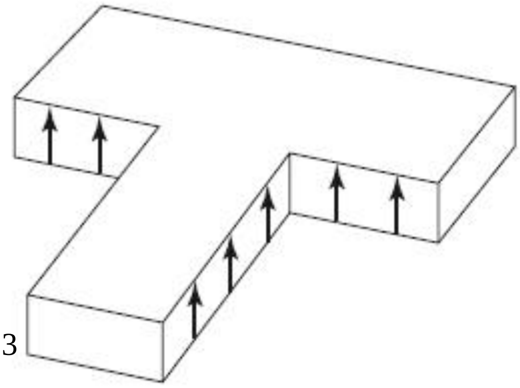
From equation 23, $b_1 = \frac{a_1}{2}$

From equation 24, $b_2 = \frac{a_1}{2}$

From equation 25, $b_3 = -\frac{a_1}{\sqrt{2}}$

Similarly we have all combinations of input and output.

4. H-Plane TEE JUNCTION (Shunt Tee):



- H-plane Tee junction is formed by cutting a rectangular slot along the width of a main waveguide and attaching another waveguide – the side arm – called as H-arm as shown in above figure 3.
- The port 1 and 2 of the main waveguide are called as collinear ports and port 3 is the H-arm or side arm.
- H-Plane Tee is so-called because the axis of the side arm is parallel to the planes of the H-field of the main transmission line. As all three arms of H-plane tee lie in the plane of magnetic field, the magnetic field divides itself into the arms; this is thus a **current junction.**
- If the H-plane junction is completely symmetrical and waves enter through the side arm, the waves that leave through the mains arms are **equal in magnitude and phase.** Since the electric field is not bent as the wave passes through a H-plane junction, but merely divides between two arms; fields of same polarity approaching the junction from the two main arms produce components of electric field that add in side arm. The effective value of field leaving through the side arm is proportional to the phasor sum of entering fields.
- Maximum energy delivery to side arm occurs when waves entering the junction through main arms are in phase. The standing wave in the main line then has an anti-node of electric field at the junction, and a current-node at the same junction. High energy delivery to a branch line connected to a transmission line at a point of high voltage and low current takes place if branch lin is connected in shunt with the main line.
- Since it is a three port junction the scattering matrix can be derived as follows:
    1. [S] Matrix of order 3 x 3.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

………………….(9)

2. Because of plane of symmetry of the junction, the Scattering coefficients are

$$S_{23} = S_{13} \dots\dots\dots\dots (26)$$

As the waves coming out of the port 1 and 2 of the collinear arm will be opposite phase and in same magnitude. Negative sign indicates phase difference.

3. If the port 3 is perfectly matched to the junction $S_{33} = 0 \dots\dots\dots\dots (27)$

4. For symmetric property $S_{ij} = S_{ji}$

$$\therefore S_{12} = S_{21}\,; \qquad S_{13} = S_{31}; \qquad S_{23} = S_{32} = S_{13} \dots\dots\dots\dots (28)$$

With the above properties, [S] becomes,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \dots\dots\dots\dots\dots\dots..(29)$$

5. From unitary property, $[S][S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1: \qquad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \dots\dots\dots (29)$$

$$R_2C_2: \qquad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \dots\dots\dots (30)$$

$$R_3C_3: \qquad 0 + |S_{13}|^2 + |S_{13}|^2 = 1 \dots\dots\dots (31)$$

$$R_3C_1: \qquad S_{13}.S_{11}^* + S_{13}.S_{12}^* = 1 \dots\dots\dots (32)$$

From equations (29), and (30), we get

$$S_{11} = S_{22} \dots\dots\dots\dots\dots..(33)$$

From equation (31), $S_{13} = \frac{1}{\sqrt{2}} \dots\dots\dots\dots\dots\dots (34)$

From equation (32), $S_{13}(S_{11}^* + S_{12}^*) = 0$

$$but\ S_{13} \neq 0 \ \therefore (S_{11}^* + S_{12}^*) = 0$$

$$\therefore S_{11}^* = -S_{22}^*$$

$$\therefore S_{11} = -S_{22} \qquad and \qquad \therefore S_{12} = -S_{11} \dots\dots\dots\dots\dots\dots (7)$$

Using these values from equation 33, 34 and 35 in equation 29,

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$\therefore S_{11} = \frac{1}{2} \ \dots\dots\dots\dots\dots\dots\dots\dots (36)$$

$$\therefore S_{12} = -\frac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)\ from\ eq.35$$

$$\therefore S_{22} = -\frac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)\ from\ eq.35$$

Substituting the values of $S_{11}, S_{12}, S_{13}, S_{22}$, the [S] matrix of equation 29 becomes

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

We know that, $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{vmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\therefore b_1 = \frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

$$\therefore b_2 = -\frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

$$\therefore b_3 = \frac{1}{\sqrt{2}}a_1 \quad \frac{1}{\sqrt{2}}a_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Case 1: Input is given at port 3 and no inputs at port 1 and 2, $a_3 \neq 0, a_1 = a_2 = 0$.

From equation 40, $b_1 = \frac{1}{\sqrt{2}}a_3$

From equation 41, $b_2 = \frac{1}{\sqrt{2}}a_3$

From equation 42, $b_3 = 0$

Let P_3 (corresponding to a_3) be the power input at port 3. Then this power divides equally between ports 1 and 2 in phase i.e. $P_1 = P_2$ (power outputs at the respective ports corresponding to b_1 and b_2. But

$$P_3 = P_1 \quad P_2 = 2P_1 = 2P_2$$

The amount of power coming out of port 1 or port 2 is due to input at port 3

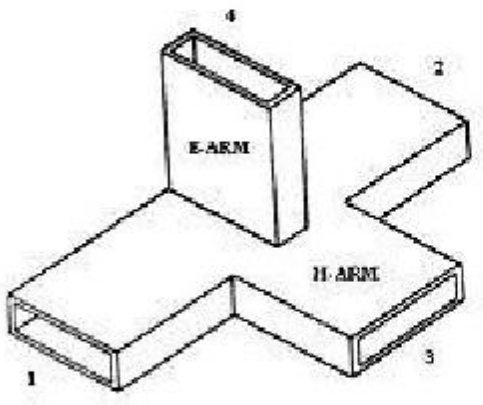
$$= 10\log_{10}\frac{P_1}{P_3} = 10\log_{10}\frac{P_1}{2P_1} = 10\log_{10}\left(\frac{1}{2}\right)$$
$$= -10\log_{10}2 = -10(0.3010) = -3\ dB$$

Hence the power coming out of the port 1 or port 2 is 3 dB down with respect to input power at port 3; hence the H-plane Tee is called as **3-dB splitter.**

Case 2: Input is given at port 1 and port 2, and no input at port 3, $a_3 = 0, \boldsymbol{a_1} = \boldsymbol{a_2} = \boldsymbol{a}$.

From equation 40, $\boldsymbol{b_1} = \frac{a}{2} - \frac{a}{2} = \boldsymbol{0}$
From equation 41, $\boldsymbol{b_2} = \frac{a}{2} - \frac{a}{2} = \boldsymbol{0}$
From equation 42, $\boldsymbol{b_3} = \frac{a}{\sqrt{2}} - \frac{a}{\sqrt{2}}$
Input at port 3 is the addition of the two inputs at port 1 and port 2 and these are added in phase.

5. **E-H Plane TEE OR MAGIC TEE:**



- A magic tee is a combination of E-plane and H-plane Tee.
- Magic tee, combines the power dividing properties of both H-plane and E-plane tee, and has the advantages of being completely matched at all the ports.
- If two signals of same magnitude and phase are fed into port 1 and port 2, then outpur will be zero at port 3 and additive at port 4.
- If signal is fed from port 4 (H-arm) then signals divides equally in magnitude and phase between port 1 and 2 and no signal appears at port 3 (E-arm).
- If signal is fed into port 3, then signal divides equally in magnitude, but opposite in phase at port 1 and 2, and no signal comes out from port 4, i.e. output at port 4 is zero.
- This magic occurs, because E-arm causes a phase delay while H-arm causes a phase advance, resulting into is $S_{12} = S_{21} = 0$.
- Using the properties of E and H-plane tee, its scattering matrix can be obtained as follows:
    1. [S] Matrix is a 4 x 4 matrix since there are 4 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \dots\dots (43)$$

2. Because of H-plane Tee junction,

$$S_{23} = S_{13} \dots\dots (44)$$

3. Because of E-plane Tee junction

$$S_{24} = -S_{14} \dots\dots (45)$$

4. Because of the geometry, an input to port 3 cannot come out of port 4 and vice versa. Hence they are called as isolated ports.

$$S_{34} = S_{43} = 0 \dots\dots. (46)$$

5. From symmetry property, $S_{ij} = S_{ji}$

$$S_{12} = S_{21} \; ; \; S_{31} = S_{13} \; ; \; S_{41} = S_{14} \; ;$$
$$S_{23} = S_{32} \; ; \; S_{34} = S_{43} \; ; \; S_{42} = S_{24} \quad \dots\dots. (47)$$

6. If ports 3 and 4 are perfectly matched to the junction.

$$S_{33} = S_{44} = 0$$

Substituting all the above results, S-matrix is

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \dots\dots\dots (48)$$

7. From unitary property, $[S][S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R_1C_1}: \quad |\boldsymbol{S_{11}}|^2 + |\boldsymbol{S_{12}}|^2 + |\boldsymbol{S_{13}}|^2 + |\boldsymbol{S_{14}}|^2 = \boldsymbol{1} \dots\dots\dots (\boldsymbol{49})$$
$$\boldsymbol{R_2C_2}: \quad |\boldsymbol{S_{12}}|^2 + |\boldsymbol{S_{22}}|^2 + |\boldsymbol{S_{13}}|^2 + |\boldsymbol{S_{14}}|^2 = \boldsymbol{1} \dots\dots\dots (\boldsymbol{50})$$
$$\boldsymbol{R_3C_3}: \quad |\boldsymbol{S_{13}}|^2 + |\boldsymbol{S_{13}}|^2 = \boldsymbol{1} \dots\dots\dots (\boldsymbol{51})$$
$$\boldsymbol{R_4C_4}: \quad |\boldsymbol{S_{14}}|^2 + |\boldsymbol{S_{14}}|^2 = \boldsymbol{1} \dots\dots\dots (\boldsymbol{52})$$

From equation 51 and 52,

$$\boldsymbol{S_{13}} = \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{2}}} \; \boldsymbol{and} \; \boldsymbol{S_{14}} = \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{2}}} \dots\dots\dots\dots\dots\dots (\boldsymbol{53})$$

Using the values of equation 53 into equation 49, we get,

$$|\boldsymbol{S_{11}}|^2 + |\boldsymbol{S_{12}}|^2 + \frac{\boldsymbol{1}}{\boldsymbol{2}} + \frac{\boldsymbol{1}}{\boldsymbol{2}} = \boldsymbol{1}$$
$$|\boldsymbol{S_{11}}|^2 + |\boldsymbol{S_{12}}|^2 = \boldsymbol{0}$$
$$|\boldsymbol{S_{11}}|^2 = -|\boldsymbol{S_{12}}|^2 \dots\dots (\boldsymbol{54})$$

Comparing equations 49 and 50, we found that $\boldsymbol{S_{11}} = \boldsymbol{S_{22}}$**……(55)**

As seen earlier $\boldsymbol{S_{12}} = \boldsymbol{S_{21}}$**=0**

$$\boldsymbol{S_{11}} = \boldsymbol{S_{12}} = \boldsymbol{S_{22}} = \boldsymbol{0}$$

This shows that port 1 and 2 are perfectly matched to the junction. Hence in any four port junction, if any tow ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction. Such a junction where in all the four ports are perfectly matched to the junction is called as MAGIC TEE.

Thus by substituting the values we get,

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \ldots\ldots\ldots (56)$$

8. We know that [b]=[S][a],

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\therefore b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

$$\therefore b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

$$\therefore b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

$$\therefore b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

Case 1: Input is given at port 3 and no inputs at port 1, 2 and 4, $a_3 \neq 0, a_1 = a_2 = a_4 = 0$.

From equation 57, $b_1 = \frac{1}{\sqrt{2}} a_3$

From equation 58, $b_2 = \frac{1}{\sqrt{2}} a_3$

From equation 59 and 60, $b_3 = b_4 = 0$

This is the property of H-plane Tee.

Case 2: Input is given at port 4 and no inputs at port 1, 2 and 3, $a_4 \neq 0, a_1 = a_2 = a_3 = 0$.

From equation 57, $b_1 = \frac{1}{\sqrt{2}} a_4$

From equation 58, $b_2 = -\frac{1}{\sqrt{2}} a_4$

From equation 59 and 60, $b_3 = b_4 = 0$

This is the property of E-plane Tee.

Case 3: Input is given at port 1 and no inputs at port 4, 2 and 3, $a_1 \neq 0, a_4 = a_2 = a_3 = 0$.

From equation 57 and 58 $b_1 = 0$, $b_2 = 0$
From equation 59 $b_3 = \frac{1}{\sqrt{2}} a_1$
From equation 60, $b_4 = -\frac{1}{\sqrt{2}} a_1$

When power is fed to port 1, nothing comes out of port 2 even though they are collinear ports (Magic!!). Hence ports 1 and 2 are called as **isolated ports.**
Similarly an input at port 2 cannot come out at port 1.
Similarly E and H-ports are **isolated ports.**

Case 4: Equal input is given at port 3 and 4; no inputs at port 1 and 2, $a_3 = a_4;\ a_1 = a_2 = 0$.

From equation 57, $b_1 = \frac{1}{\sqrt{2}}(2a_3)$,
From equation 58, 59 and 60, $b_2 = b_3 = b_4 = 0$
This is called as an additive property.

Case 5: Equal input is given at port 1 and 2; no inputs at port 3 and 4, $a_1 = a_2;\ a_3 = a_4 = 0$.

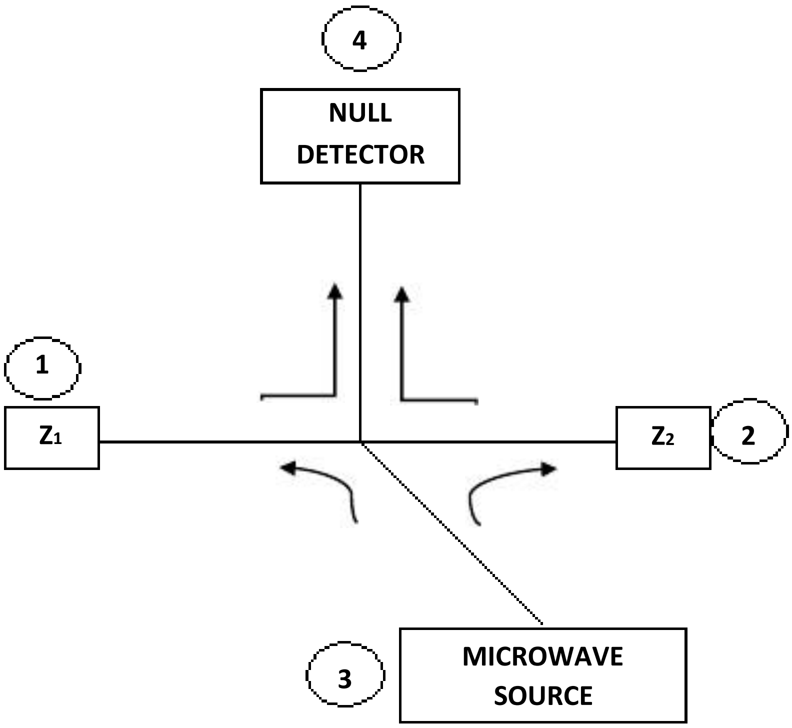
From equation 57, 58, and 60, $b_1 = b_2 = b_4 = 0$
From equation 59, $b_3 = \frac{1}{\sqrt{2}}(2a_1)$

Equal inputs at ports 1 and 2 results in an output port 3 (additive port)and no output at port 1, 2 and 4. This is similar to case 4.

Applications of magic tee:

1. **Measurement of Impedance:**
   Magic tee has been used in the form of a bridge, as shown in figure below for measuring impedance.
   Microwave source is connected in arm 3
   A null detector is connected in arm 4.
   The unknown impedance is connected at arm 2.
   Standard variable known impedance is connected in arm 1.

   Using the properties of magic tee, power from port 3 divides equally in port 1 and 2.



Now known impedance Z1 and unknown impedance Z2 is not equal to characteristic impedance Z0. Hence there will be reflections from port 1 and 2 towards the junction.

If ρ_1 and ρ_2 are reflection coefficients, then

The reflection from port 1 is $\frac{\rho_1 a_3}{\sqrt{2}}$

The reflection from port 2 is $\frac{\rho_2 a_3}{\sqrt{2}}$

The resultant wave reaching at null port i.e. at port 4 is,

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\rho_1 a_3\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\rho_2 a_3\right)$$

$$= \frac{1}{2}(\rho_1 a_3 - \rho_2 a_3)$$

For perfect balancing,

$$\frac{1}{2}(\rho_1 a_3 - \rho_2 a_3) = 0$$

$$\therefore \ \rho_1 = \rho_2$$

But $\rho_1 = \frac{Z_1 - Z_2}{Z_1 - Z_2}$ and $\rho_2 = \frac{Z_2 - Z_2}{Z_2 + Z_2}$

$$\therefore \ \frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{Z_2 - Z_2}{Z_2 + Z_2}$$

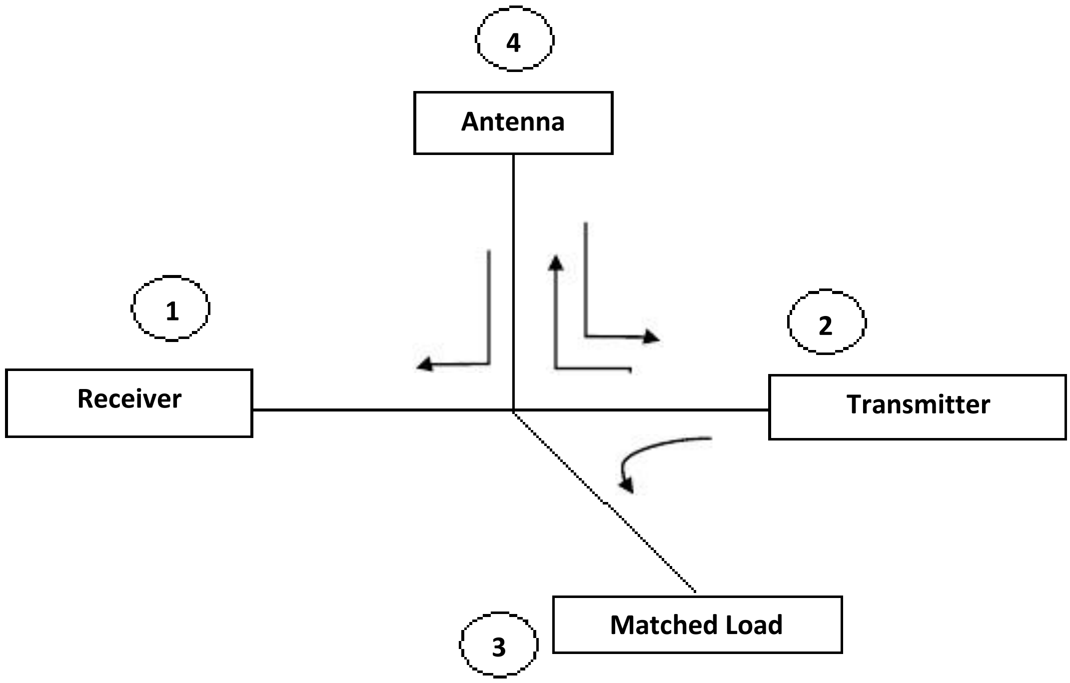
Or

$$Z_1 = Z_2$$

Thus unknown impedance can be measured by adjusting the standard variable impedance till the bridge is balance and both impedances become equal.

2. Magic tee as a Duplexer:

- In magic tee, port 1 and 2 are isolated ports, and the same property is used to isolate sensitive receiver from high power transmitter.
- The transmitter is connected to port 2 and receiver is connected to port 1, antenna at port 4 i.e. E-arm and matched load at port 3 i.e. H-arm.
- During transmission half power reaches to the antenna from where it is radiated inot space.
- Other half power reaches to the matched load where it is absorbed without any reflections.
- No transmitter power reaches the receiver since port 1 and 2 are isolated ports in Magic Tee.

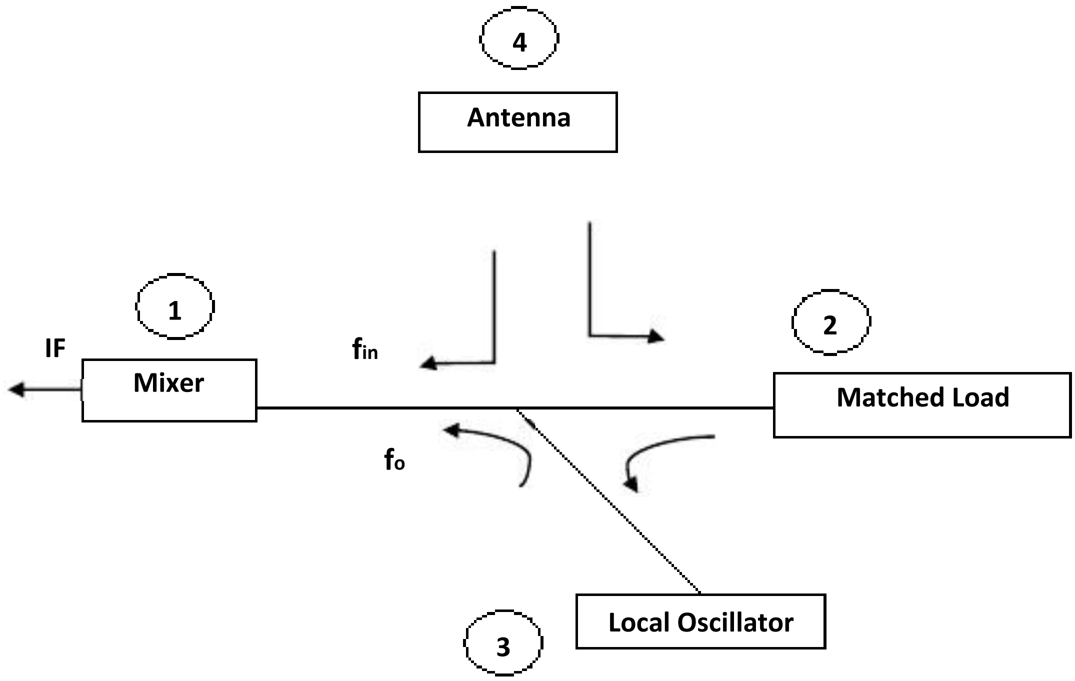


3. Magic tee as a Mixer:

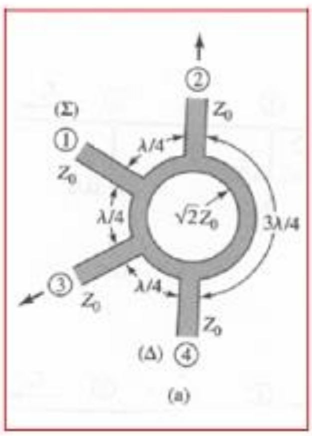
- A magic tee can also be used in microwave receivers as a mixer where the signal and local oscillator are fed into the E and H arm as shown in figure below.
- Half of the local oscillator power and half of the received power from antenna goes to the mixer where they are mixed to generate the IF frequency.

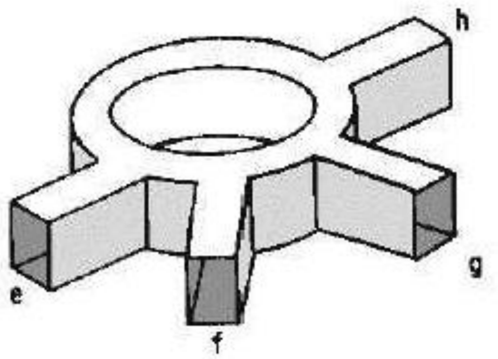
$$IF = f_s \sim f_0$$

- Magic tee has many other applications such as microwave discriminator, Microwave Bridge, etc.



6. **Hybrid Ring:**





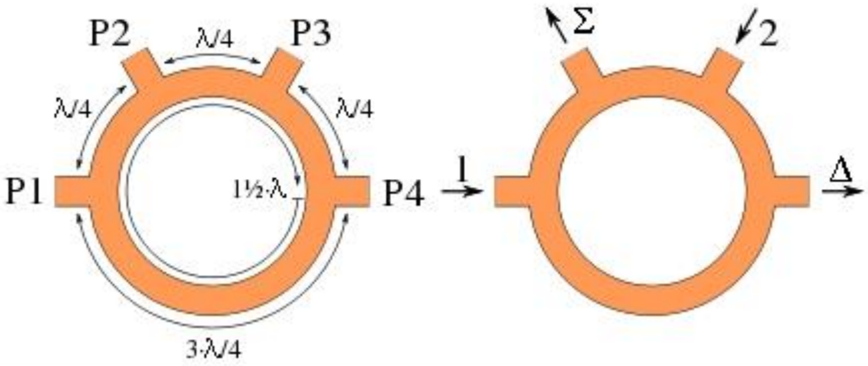
(A)

- Rat race (Ring hybrid) is one of the oldest and simplest designs for the fabrication of a 180° hybrid.
- As shown in above figure, it is a ring shape making transmission lines which compose of three λ/4 line sections and one 3λ/4 line section
- To describe the operation, if port 1 is excited, the waves will be transmitted towards the neighboring ports, port 2 and port4, equally. The other port is isolated. Two identical waves are transmitted in clockwise and anti-clockwise direction respectively such that the waves are 180 ° out of phase at the interacting port 3. So the voltages are cancelled out and become zero at this point. The isolated port lets the circuit become a three-port network. Due to

the impedance of the rat-race ring being constant, the voltages are split equally to port 2 and port4. However the phase is not identical because the path from port 1 to port 2 is one-half wavelength which is longer than the path from port 1 and port 4 is 180 : To infer, a table is constructed to illustrate the situation when different ports are excited.

Excited Port	Output Port	Isolated Port	Phase difference between two outpu ports
1	2 , 4	3	180°
2	1 , 3	4	180°
3	2 , 4	1	0°
4	1 , 3	2	0°

Table 1 Conventional rat-race hybrid operation



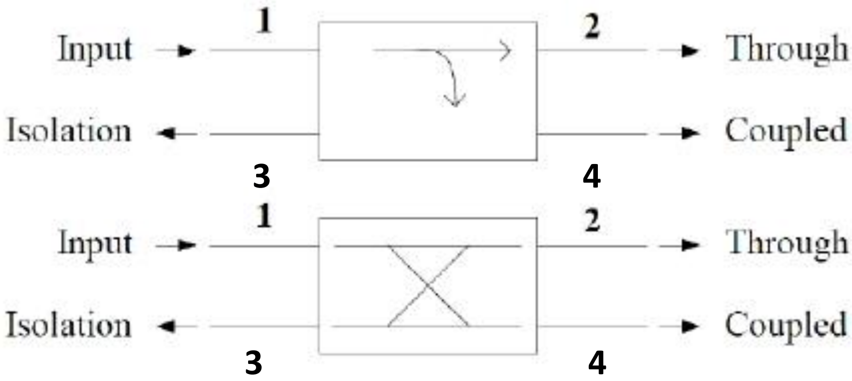
The scattering matrix can be written as,

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

7. Directional Coupler:

- Directional couplers are flanged, built in waveguide assemblies which can sample a small amount of microwave power for measurement purposes.
- The directional couplers are passive devices used in the field of radio technology.
- These devices fit for the power transmitted through a transmission line to another port using two transmission lines placed close enough so that the energy flowing through one of the lines are coupled to each other.

- Directional couplers are defined to be passive microwave components used for power division.
- During the whole process of power division we can notice that the four-port networks take the form of directional couplers and hybrids.
- While directional couplers can be created having in mind the arbitrary power division, the hybrid junctions have frequently identical power division.
- Regarding the hybrid junctions we can take in consideration two situations: a 90° (quadrate) or a 180° (magic-T) phase shift between the output ports.
- Looking back at the important steps that have been taken in the **HISTORY**, is important to mention that at the MIT Radiation Laboratory in the 1940s, were invented and characterized a diversity of waveguide couplers, including E- and H-plane waveguide tee junctions, the Bethe hole coupler, multihole directional couplers, the Schwinger coupler, the waveguide magic-T, and several types of couplers using coaxial probes. Another important phase in development of the couplers is the period between 1950s and 1960s, when it took place a reinvention of a lot of them to use stripline or microstrip technology. New types of couplers, like the branch line hybrid, and the coupled line directional coupler also had benefit of a development, due to the expanding use of planar lines.
- **Directional couplers characterization :** A figure below illustrates the basic operation of a directional coupler:



- A directional coupler has four ports:
  - o P_i: incident power at port 1.
  - o P_r: received power at port 2.
  - o P_f: forward coupled power at port 4.
  - o P_b: back power at port 3.
- Directional coupler are built in waveguide assemblies, used to sample a small amount of microwave power for measurement purposes, and can be either unidirectional on (i.e. measuring only the incident power) or bi-directional one (measuring both incident power and reflected power).
- With matched terminations at all ports, the properties of an ideal directional coupler can be summarized as follows:
  - o A portion of power travelling from incident port to received port is coupled to coupling port but not to isolation port .
  - o A portion of power travelling from incident port to received port is coupled to isolation port but not to coupling port (bi-directional case).

- o A portion of power incident on isolation port is coupled to receive port but not to incident port and a portion of power incident on coupling port is coupled to incident port but not to received port. Also incident and isolated ports are decoupled as are received and coupled ports.

- **Coupling factor, C:** it is defined as the ratio of the incident power P_i to the forward power P_f measured in dB.

$$C = 10\log_{10}\frac{P_i}{P_f}\ dB$$

- **Directivity, D:** the directivity of a D.C. is defined as the ratio of forward power P_f to the back power P_b, expressed in dB.

$$D = 10\log_{10}\frac{P_f}{P_b}\ dB$$

- Coupling factor is a measure of how much of the incident power is being sampled while directivity is the measure of how well the directional coupler distinguishes between the forward and reverse travelling powers.
- **Isolation, I:** it is defined to describe the directive properties of a directional coupler. It is defined as the ratio of incident power P_i to the back power P_b.

$$I = 10\log_{10}\frac{P_i}{P_b}\ dB$$

- Isolation in dB is equal to the coupling factor plus directivity.
- As with any component or system, there are several specifications associated with RF directional couplers. The major RF directional coupler specifications are summarized in the table below.

Term	**Description**
Coupling Loss	Amount of power lost to the coupled port (3) and to the isolated port (4). Assuming a reasonable directivity, the power transferred unintentionally to the isolated port will be negligible compared to that transferred intentionally to coupled port.
Main line loss	Resistive loss due to heating (separate from coupling loss). This value is added to the theoretical reduction in power that is transferred to the coupled and isolated ports (coupling loss).
Directivity	Power level difference between Port 3 and Port 4 (related to isolation). This is a measure of how independent the coupled and isolated ports are. Because it is impossible to build a perfect coupler, there will always be some amount of unintended coupling between all the signal paths.
Isolation	Power level difference between Port 1 and Port 4 (related to directivity).

SCATTERING MATRIX OF DIRECTIONAL COUPLER

1 Directional coupler is a 4-port network. Hence [S] is 4 x 4 matrix.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

2 In a directional coupler all four port are perfectly matched to the junction. Hence the diagonal elements are zero.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

3 From symmetry property, $S_{ij} = S_{ji}$

$$S_{31} = S_{13}\ ;\ S_{41} = S_{14}\ ;\ S_{23} = S_{32}\ ;\ S_{34} = S_{43}\ ;\ S_{42} = S_{24}$$

Ideally back power is zero (P_b=0) i.e. there is no coupling between port 1 and 2.

$$\therefore\ S_{13} = S_{31} = 0$$

4 Also there is no coupling between port 2 and port 3.

$$\therefore\ S_{24} = S_{42} = 0$$

Substituting the above values of scattering parameters into S-matrix, we get,

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

5 From unitary property $[S][S]^* = [I]$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R_1C_1}: \quad |\boldsymbol{S_{12}}|^2 + |\boldsymbol{S_{14}}|^2 = \boldsymbol{1} \ldots\ldots\ldots (\boldsymbol{61})$$
$$\boldsymbol{R_2C_2}: \quad |\boldsymbol{S_{12}}|^2 + |\boldsymbol{S_{23}}|^2 = \boldsymbol{1} \ldots\ldots\ldots (\boldsymbol{62})$$
$$\boldsymbol{R_3C_3}: \quad |\boldsymbol{S_{23}}|^2 + |\boldsymbol{S_{34}}|^2 = \boldsymbol{1} \ldots\ldots\ldots (\boldsymbol{63})$$
$$\boldsymbol{R_1C_3}: \quad \boldsymbol{S_{12}}.\boldsymbol{S_{23}}^* + \boldsymbol{S_{14}}.\boldsymbol{S_{34}}^* = \boldsymbol{0} \ldots\ldots\ldots (\boldsymbol{64})$$

Comparing equations 61, and 62, and 62 and 63, we get

$$S_{14} = S_{23};$$

$$S_{12} = S_{34}$$

Let assume that, S_{12} is real and positive = 'P'

$$S_{12} = S_{34} = P = S_{34}^*$$

$$S_{12}.S_{23}^{*} \quad S_{14}.S_{34}^{*} = 0$$

$$P.S_{23}^{*} - S_{14}.P = 0$$

$$P \neq 0; \quad S_{23}^{*} + S_{14} = 0$$

$$S_{23} = jq; \; S_{14} = -jq$$

$$i.e.\, S_{23} = S_{14} \;\; must\ be\ an\ imaginary\ parameter.$$

Hence S-matrix of a directional coupler is reduced to

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$